MATH 1A − QUIZ 10 − SOLUTIONS

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Name:____

Instructions: You have 12 minutes to take this quiz, for a total of 10 points. **Show your work!** May your luck be integrable!

(1) *(4 points)* Using **the definition of the integral**, evaluate:

$$\int_2^5 (2-x)^3 \, dx$$

First of all $\Delta x = \frac{5-2}{n} = \frac{3}{n}$, and $x_i = 2 + \frac{3i}{n}$. Therefore:

$$\begin{aligned}
\int_2^5 (2-x)^3 \, dx &= \lim_{n\to\infty} \sum_{i=1}^n \frac{3}{n}\left(2 - \left(2 + \frac{3i}{n}\right)\right)^3 \\
&= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^n \left(\frac{-3i}{n}\right)^3 \\
&= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^n \left(\frac{(-27)i^3}{n^3}\right) \\
&= \lim_{n\to\infty} \frac{-81}{n^4} \sum_{i=1}^n i^3 \\
&= \lim_{n\to\infty} \frac{-81}{n^4} \left(\frac{n^2(n+1)^2}{4}\right) \\
&= -\frac{81}{4} \lim_{n\to\infty} \frac{n^2(n+1)^2}{n^4} \\
&= -\frac{81}{4}
\end{aligned}$$

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(2) *(4 points)* Calculate the following limit.

Note: The trick is to fish out a $\frac{1}{n}$, then to write this in terms of $\frac{i}{n}$, and to recognize that limit as an integral (and evaluate it)

$$\begin{aligned}
&\lim_{n\to\infty} \frac{n}{n^2+1} + \frac{n}{n^2+4} + \frac{n}{n^2+9} + \cdots + \frac{n}{n^2+n^2} \\
=&\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{n^2+i^2} \\
=&\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{n^2}{n^2+i^2}\right) \\
=&\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{n^2}{n^2\left(1+\frac{i^2}{n^2}\right)}\right) \\
=&\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{1+\left(\frac{i}{n}\right)^2}\right) \\
=&\int_0^1 \frac{1}{1+x^2} dx \\
=&\left[\tan^{-1}(x)\right]_0^1 \\
=&\tan^{-1}(1) - \tan^{-1}(0) \\
=&\frac{\pi}{4} - 0 \\
=&\frac{\pi}{4}
\end{aligned}$$

(3) *(2 points)* Find the derivative of the following function:

$$f(x) = \int_{\sin(x)}^{1+e^x} e^{-t^2} dt$$

Let F be an antiderivative of e^{-t^2}, then $f(x) = F(1+e^x) - F(\sin(x))$, so:

$$\begin{aligned}
f'(x) =& F'(1+e^x)e^x - F'(\sin(x))\cos(x) \\
=& f(1+e^x)e^x - f(\sin(x))\cos(x) \\
=& e^{-(1+e^x)^2} e^x - e^{-\sin^2(x)} \cos(x)
\end{aligned}$$